DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics Semester-V

DISCIPLINE SPECIFIC CORE COURSE – 13: METRIC SPACES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Metric Spaces	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

Learning Outcomes: This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities. Become aware one such formulations leading to metric spaces.
- Analyse how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.
- Know about Banach fixed point theorem, whose far-reaching consequences have resulted into an independent branch of study in analysis, known as fixed point theory.

SYLLABUS OF DSC-13

UNIT – I: Topology of Metric Spaces (18 hours)

Definition, examples, sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor's theorem, Subspaces.

UNIT – II: Continuity and Uniform Continuity in Metric Spaces (15 hours)

Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity; Homeomorphism, Isometry and equivalent metrics, Contraction mapping, Banach fixed point theorem.

UNIT – III: Connectedness and Compactness (12 hours)

Connectedness, Connected subsets of $\mathbb{R}$, Connectedness and continuous mappings, Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

Essential Reading

3. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

Suggestive Readings

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. New Delhi.
- Rudin, Walter. Principles of mathematical Analysis (3rd ed.).
- Simmons, George F. (2004). Introduction to Topology and Modern Analysis. McGraw-Hill Education. New Delhi.

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

DISCIPLINE SPECIFIC CORE COURSE – 14: RING THEORY

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Ring Theory	4	3	1	0	Class XII pass with Mathematics	DSC-7: Group Theory

Learning Objectives: The primary objective of this course is to:

- Introduce the fundamental theory of rings, and their homomorphisms.
- Develop the basic concepts of polynomial rings and irreducibility tests for polynomials over the ring of integers, and rational numbers.
- Introduce polynomial analog of a prime number.
- Describe polynomial rings, principal ideal domains, Euclidean domains and unique factorization domains, and their relationships.

Learning Outcomes: This course will enable the students to:

- Learn about the fundamental concept of rings, integral domains, and fields.
- Know about ring homomorphisms and isomorphisms theorems of rings, and construct quotient fields for integral domains.
- Appreciate the significance of unique factorization in rings and integral domains.
- Apply several criteria for determining when polynomials with integer coefficients have rational roots or are irreducible over the field of rational numbers.

SYLLABUS OF DSC-14

UNIT – I: Introduction to Rings and Ideals (18 hours)

Definition and examples of rings, Properties of rings, Subrings, Integral domains and fields, Characteristic of a ring; Ideals, operations on ideals, ideal generated by a set and properties, Factor rings, Prime ideals and maximal ideals, Principal ideal domains.

UNIT – II: Ring Homomorphisms and Polynomial Rings (15 hours)

Definition, examples and properties of ring homomorphisms; First, second and third

isomorphism theorems for rings; The field of quotients; Polynomial rings over commutative rings, Division algorithm and consequences.

UNIT–III: Unique Factorization Domain and Divisibility in Integral Domains (12 hours)
Factorization of polynomials, Reducibility tests, Mod *p* Irreducibility test, Eisenstein's criterion, Unique factorization in $\mathbb{Z}[x]$; Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
2. Dummit, David S. & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.

Suggestive Readings

- Herstein, I. N. (2006). Topics in Algebra (2nd ed.). Wiley Student Edition. India.
- Hungerford, Thomas W. (2012). Abstract Algebra: An Introduction (3rd ed.). Cengage Learning.

DISCIPLINE SPECIFIC CORE COURSE – 15: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Partial Differential Equations	4	3	0	1	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Basic concepts of first and second order linear/nonlinear partial differential equations.
- Modeling of wave equation, heat equation, Burgers equation, traffic flow and their solutions.

Learning Outcomes: The course will enable the students to learn:

- The method of characteristics and reduction to canonical forms to solve first and second order linear/nonlinear partial differential equations.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.
- The Cauchy problem and solutions of wave equations with initial boundary-value problems, and non-homogeneous boundary conditions.

SYLLABUS OF DSC-15

UNIT – I: First Order Partial Differential Equations (15 hours)
Basic concepts, classification, construction, and geometrical interpretation; Method of characteristics and general solutions, Cauchy problem for a first-order PDE, Canonical

forms of first-order linear equations; Method of separation of variables; Charpit's method for solving non-linear PDEs.

UNIT – II: Classification and Solutions of Second-Order Linear PDEs (12 hours)

Classification (hyperbolic, parabolic, and elliptic), reduction to canonical forms, and general solutions of second-order linear PDEs; Higher order linear partial differential equations with constant coefficients.

UNIT – III: Applications of Partial Differential Equations (18 hours)

Mathematical models: The vibrating string, vibrating membrane, conduction of heat in solids, the gravitational potential, conservation laws and the Burgers equation, Traffic flow; Cauchy problem and wave equations: Solutions of homogeneous wave equations with initial boundary-value problems, and non-homogeneous boundary conditions, Cauchy problem for non-homogeneous wave equations.

Essential Readings

1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.

2 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

Suggestive Readings

- Abell, Martha & Braselton, J.P. (2004) Differential Equations with Mathematica, Elsevier, Academic Press, Third Edition.
- Stavroulakis, Ioannis P & Tersian, Stepan A. (2004). Partial Differential Equations: An Introduction with Mathematica and MAPLE (2nd ed.). World Scientific.

Practical (30 hours)- Practical / Lab work to be performed in a Computer Lab:

Modeling of the following similar problems using SageMath/Python/Mathematica/ MATLAB/Maple/Maxima/Scilab:

1. General solution of first and second order partial differential equations.
2. Solution and plotting of Cauchy problem for first order PDEs.
3. Plotting the characteristics for the first order partial differential equations.
4. Solution of vibrating string problem using D'Alembert formula with initial conditions.
5. Solution of heat equation $u_t = k\, u_{xx}$ with initial conditions.
6. Solution of one-dimensional wave equation with initial conditions:
   i. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ x \in \mathbb{R}, t > 0$
   ii. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ u(0,t) = 0,\ x \in \mathbb{R}, t > 0$
   iii. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ u_x(0,t) = 0,\ x \in \mathbb{R}, t > 0$
7. Solution of traffic flow problem with given initial conditions, and plotting of the characteristic base curves and the traffic density function.

B.Sc. (Hons) Mathematics, Semester-V, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(i): MATHEMATICAL DATA SCIENCE

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title &	Credits	Credit distribution of the course	Eligibility	Pre-requisite of